

No Calculators

Problems 13-14. Time limit: 10 minutes.

13. On the planet Triad there are twelve cities. Every set of three cities is connected by one triangular road. How many triangular roads are there on Triad?

14. The symbol  $\vee$  means “or”, and the symbol  $\underline{\vee}$  means “exclusive or”(true if one or the other is true, but not both). Consider the two expressions  $(p \vee q) \underline{\vee} r$  and  $(p \underline{\vee} q) \vee r$ . Of the eight cases, how many have different truth values?

Problems 15-16. Time limit: 10 minutes.

15. For all real numbers  $x$ ,  $2f(x) + f(6 - x) = x^2$ . Find the value of  $f(3)$ .

16. Find the exact volume of a cube if the space diagonal (longest diagonal; connecting opposite vertices of a cube) has length 18.

Problems 17-18. Time limit: 10 minutes.

17. Find the equation (in slope-intercept form) of the line tangent to the circle with equation  $x^2 + (y - 2)^2 = 25$  at the point  $(3, 6)$ .

18. How many five-digit numbers can be made from the set of digits  $\{2, 3, 4, 5, 6, 7, 8, 9\}$  with the following rules:  
 (a) no digit can be repeated;  
 (b) the digits must be in decreasing order?

Answers.

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|-----|------------------------------------|-----|---------------|
| 13. | 220                                | 14. | 4             |
| 15. | 3                                  | 16. | $648\sqrt{3}$ |
| 17. | $y = \frac{-3}{4}x + \frac{33}{4}$ | 18. | 56            |