Nassau County Interscholastic Mathematics League

Solutions **2001-2002**

Contest # 6

25. f(x) = x, so $|3x-1| = x$, *so* $3x-1 = x$ *for* $x \geq \frac{1}{3}$ and $3x-1=-x$ *for* $x<\frac{1}{3}$. In the first case $x=\frac{1}{2}$ and in the second, $x=\frac{1}{4}$. Both are non-negative.

26. If the house is blue, then all three statements are true. If the house is red, then Andy's statement is true, Bill's is true, and Colleen's is false. So it's red.

27. Look at ΔABD. It contains a 40 ° angle and a right angle, so $m\angle BAE = 50°$. $m\overset{\frown}{BE} =$ 2(measure of inscribed $\angle$) $= 100°$.

28. Let $x=y=0$ and get $f(0) = 2f(0)g(0)$ *and* $g(0) = (g(0))^2 - (f(0))^2$. Now either $f(0)=0$ or $g(0)=\frac{1}{2}$. If $f(0)=0$, then $g(0)=(g(0))^2$, so $g(0)=0$ *or* $g(0)=1$. If $g(0)=\frac{1}{2}$, then $f(0)$ is not real.

29. $(x+y)(x-y) = x-y$ so $x+y=1$ *or* $x-y=0$. So $x=y$ *or* $x=1-y$. If $x=y$, then $x=0$. So $x=1-y$. Substituting, $(1-y)y = 1-y-y \rightarrow y-y^2 = 1-2y \rightarrow y^2-3y+1=0$. Solving, y = $\frac{3-\sqrt{5}}{2}$ is the only positive solution and, substituting, x = $\frac{-1+\sqrt{5}}{2}$.

30. Method 1: ΔABC *has area* $\frac{\sqrt{3}}{4}$. So ΔBEC has half that area, $\frac{\sqrt{3}}{8}$. BG:BE = 2:3, so area ΔBDC:areaΔBEC = 4:9. So DGEC is 5/9 of ΔBEC. The area is $\frac{5}{9}\cdot\frac{\sqrt{3}}{8}=\frac{5\sqrt{3}}{72}$

Method 2: Draw the median from A, intersecting $\overline{CB}$ at Q. The area of the equilateral triangle is $\frac{\sqrt{3}}{4}$, so the area of $\Delta EGC = \frac{1}{6}\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{24}$, using the theorem that the 6 triangles formed by the medians have equal area in any Δ. $BG:GE = 2:1$ and $\overline{GD} \parallel \overline{EC}$, so $BD:CD = 2:1$ by the side splitter theorem. $DC = \frac{1}{3}$ and $QD = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$. That means $DC:QD = 2:1$. So the area of $\Delta DGC = \frac{2}{3}\cdot\frac{\sqrt{3}}{24} = \frac{\sqrt{3}}{36}$. Adding $\frac{\sqrt{3}}{36}+\frac{\sqrt{3}}{24}=\frac{5\sqrt{3}}{72}$.