

Solutions
Contest # 3

13. $(2x)(x) - (x)(1) = 3$. So we get a quadratic $2x^2 - x - 3 = 0$. This factors into $(2x + 3)(x - 1) = 0$ So $x = -1$ or $\frac{3}{2}$

14. Use the angle bisector theorem on $\triangle DBC$ to get $\frac{8}{DE} = \frac{6}{3}$ so $DE = 4$ Now use the theorem and $DC = 7$. on $\triangle ADC$ to get $\frac{AD}{4} = \frac{7}{6}$ so $AD = \frac{14}{3}$

15. Let the leg be x and the hypotenuse $x + 2$. So $x^2 + 20^2 = (x + 2)^2$. Solve to get $x = 99$ and the hypotenuse is 101.

16. If they were all distinct, there would be $5!$ (5 "factorial"), but for the two pairs of identical letters we have to divide by $2!$ twice, so $\frac{5!}{(2!)(2!)} = 30$ [this idea is likely to show up again]

17. Since $(a + b)(a - b) = 37$, and a, b are both positive integers, and 37 is prime, we must have $a + b = 37$
 $a - b = 1$
so $a = 19$ and $b = 18$.

18. The number we seek must be one less than a common multiple of 2, 4, 6, and 8 (hence of the LCM = 24), but 2 less than a multiple of 11. Trying numbers one less than multiples of 24, i.e. 23, 47, 71, 95, 119, ..., we find that 119 is the first to be 2 less than a multiple of 11.