Nassau County Interscholastic Mathematics League

Solutions **2002-2003**
Contest # 5

19. Expanding the determinant we get, $3(40-2x)-x(20-x)+7(8-8)=0$, which simplifies to $x^2-26x+120=0,$ and then $x=6$ *or* 20

20. $6272=2^7\cdot 7^2$, so $(a.b)=(2,7)$

21. Divide and rewrite $\frac{n^2+n}{n+4}=n-3+\frac{12}{n+4}$ so to be an integer n+4 divides into 12, but $n>0$, so $n+4>4$, and thus $n+4=6$ or 12, and n = 2 or 8.

22. AB = CD = 12, so BC = AD = 10. Since the altitude of 5 could go either way, set the areas equal to get (10)(5) = (12)(height) or (12)(5) = (10)(height) so the other height could be either 6 or $\frac{25}{6}$

23. Since he ends up in his original location he must have done 2 steps each way, but in any order, so the probability is $({}_4C_2)\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2=\frac{8}{27}$

24. The vertices of the triangle are (0,0), (0,12), and $\left(\frac{-12}{m},0\right)$. The perimeter will be

$12+\frac{12}{m}+\sqrt{12^2+\left(\frac{12}{m}\right)^2}=36$. Let $\frac{12}{m}=x$ and the equation becomes

$x+\sqrt{144+x^2}=24\,so\,\sqrt{144+x^2}=24-x$. Squaring both sides and simplifying we get 48x=132, so x = 9.

Since $\frac{12}{m}=9,\ m=\frac{12}{9}=\frac{4}{3}$

Alternatively, the leg containing the y axis has length 12. If the other leg is represented by x, the hypotenuse is $24-x$. By the Pythagorean theorem:

$x^2+12^2=(24-x)^2$

$x^2+144=24^2-48x+x^2$,

$x=9$

making the slope $\frac{12}{9}=\frac{4}{3}$