Nassau County Interscholastic Mathematics League

Solutions **2002-2003**

Contest # 6

25. There are 130 black, 100 blue and so 139 other. Pick two. Probability that one is black, one is other $\frac{{}_{130}C_1 \cdot {}_{139}C_1}{{}_{369}C_2} \approx 0.266$

26. Divide to get $\frac{n^2+4}{n+5} = n-5+\frac{29}{n+5}$ which will be an integer if n+5 divides 29. So n+5 = 1 or 29 or –1 or -29. So $n = -4$ or n = 24 or n = -6 or n = -34

27. A can't be a Gilliam (they don't speak first). If A were a Grail, then so is B, but then B lies, so A can't be a Grail. Hence A must be a Graham. Now B can't be a Grail. Can B be a Gilliam. If so, he's telling the truth, but that's impossible. So B is a Graham. Finally, C lied so he must also be a Graham. So there are no Grails.

28. There are three choices for first and last. Then there are nine interior letters, two pairs of which are indistinguishable, so the number of ways will be $3 \cdot \frac{9!}{(2!)(2!)} = 272160$

29. Let the consecutive integers be n and n+1. $(n+1)^2 - n^2 = 2n+1$ must be an odd number, but since n>0, it can't be 1. So all the odd numbers from 3 to 499 work and there are 249 of them.

30. This little known theorem is called the "Broken Chord Theorem." In general, AD + DE = BE so BE = 7. There are several proofs (none very easy). One method is to mark F on EB so that DE = EF. Then prove that $\Delta CED \cong \Delta CEF$ and $\Delta CDA \cong \Delta CFB$ Now AD = FB and DE = EF. Thus AD + DE = FB + EF = EB.

Alternative, extend $\overline{CE}$ to the other side of the circle. Draw a line from A parallel to $\overline{DB}$. CE=a, and the corresponding piece from the parallel to the circle is also a. The segment between these two pieces is 5. Let BE=x. By the power theorem for chords, $2x = a(5+a)$. By congruent triangles, $5+2a = 2+x$ (The longer legs of the triangles with diameters as hypotenuses are congruent) Solve for a $a = \frac{x-3}{2}$ and substitute in the power equation $2x = \left(\frac{x-3}{2}\right)\left(5+\frac{x-3}{3}\right)$, simplify to $x^2-4x-21=0$ so $x=7$ or $x=-3$. Reject -3, EB=7