Nassau County Interscholastic Mathematics League

Solutions **2002-2003**

Contest # 2

7. There can't be 0 or 4 or 1 or 3 guilty. So there are 2 guilty.

8. Since $\overline{BC}$ bisects $\angle$ABD, we have the proportion $\frac{AB}{AC}=\frac{BD}{CD}$. Let AC = x, so CD = 10 – x, and now $\frac{6}{x}=\frac{9}{10-x}$, and $9x=60-6x$, so $x=4$ [this proportion theorem is likely to show up again]

Alternative solution: Since AB:BD=6:9=2:3, let AC=2x, CD=3x. 5x=10, x=2, 2x=4

9. Let the fourth root be r, so the function is of the form $f(x)=(-2)(x-4)(x+3)\left(x-\frac{1}{2}\right)(x-r)$

But the point $(0,48)$ is on the graph, so plug in to get the equation $(-2)(-4)(3)\left(-\frac{1}{2}\right)(-r)=-48$.

So $12r=-48$ and $r=-4$

10. We have the equations $\frac{a+b}{2}=c$, $\frac{a+c}{2}=b+1$, and $\frac{b+c}{2}=2a+2$ so $a+b=2c, a+c=2b+2$, and $b+c=4a+4$. From equation 1, a = 2c – b, substitute into the second and third equations and simplify to get 3b = 3c – 2 and 5b = 7c + 4. Solving, we get $c=-\frac{11}{3}$. Alternatively, use matrices on your calculator to solve the system.

11. Let x = number of people at the party. Each person shakes hands with x – 2 people, and so the total number of handshakes would be $\frac{x(x-2)}{2}$ [note: divide by 2 so each handshake is counted once]. Set this equal to 40 and x = 10, so there are 5 couples.

12. The number we seek is exactly one less than a multiple of 8, one less than a multiple of 6, one less than a multiple of 4, and one less than a multiple of 2. So it's one less than the least common multiple of 8, 6, 4, and 2, which is 24. So the least number is 23. [this idea is likely to show up again] Alternatively use modular arithmetic (is that really alternatively or just different notation for the same idea?)