

Solutions  
Contest # 2

7. Solve the equation  $2x^2 - 7x - 15 = 0$ ,  $(2x + 3)(x - 5) = 0$ ,  $x = -\frac{3}{2}$  or  $x = 5$ . The integers which satisfy the inequality are the ones between  $-\frac{3}{2}$  and 5, including 5. There are 7 of them.

Alternate solution: graph the parabola  $y = 2x^2 - 7x - 15$  and find the integers for which the graph is below or on the x-axis. They are  $-1, 0, 1, 2, 3, 4$ , or 5. So there are 7.

8. The radius of the circle is 8, so each side is  $8\sqrt{2}$  and the area is the square of the side, which is **128**.

Alternate solution. The diameter is 16. The area of a quadrilateral with perpendicular diagonals is half the product of the diagonals.  $\frac{1}{2} \cdot 16 \cdot 16 = \mathbf{128}$ .

9. One way to tell if a function is even is to look at the graph and see if it's symmetric over the y-axis. Another test would be to try some opposite inputs and see if the same result occurs. For example,  $\log_2(-5)$  is not the same as  $\log_2(5)$  so  $\log_2$  is not even. Of the functions given, the even ones are  $\cos(x)$ ,  $5x^2 + 3$ , and  $|x|$ . There are **4**.

10. Let the base be b. So  $(2b^2 + 5b + 7)(9) = b^3 + 6b^2 + 4b + 3$ . The only real solution is  $b = \mathbf{15}$ .

Alternate solution: multiplying the units digits  $(9)(7)$  ends in 3, so the remainder when 63 is divided by the base is 3. So the base is a multiple of 5. Also it must be greater than 9, since 9 is one digit. Try 10, 15, etc. until one works. Here **15** works.

11. The probability the first box is acceptable is 1 since it can be anything. The next must be different, so the probability is  $\frac{5}{6}$ . The third must be again different, probability  $\frac{4}{6}$ , etc. So the probability they are all different

$$\text{is } \left(\frac{6}{6}\right)\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{6}\right) = \frac{\mathbf{5}}{\mathbf{324}} \approx \mathbf{0.0154}.$$

12. First note that the triangle is a right triangle since  $18^2 + 80^2 = 82^2$ . Compute the area two ways; either half the product of the legs or half the product of the hypotenuse and the altitude to the hypotenuse (the shortest). So

$$(18)(80) = (82)(\text{alt}), \text{ and the altitude} = \frac{\mathbf{720}}{\mathbf{41}} \approx \mathbf{17.561}.$$