Nassau County Interscholastic Mathematics League

Solutions **2003-2004**
Contest # 4

T1. Using the Change of Base Theorem on the left side, $\log_3 y = 2$, *so* $y = \mathbf{9}$.

T2. Since $\overline{AB}$ is a diameter, it is perpendicular to $\overline{AD}$ and $\overline{CB}$, so $\overline{AB}$ is the altitude of the trapezoid. The area of the trapezoid is $\frac{1}{2}(AB)(AD+BC)$. By the two tangent theorem, $AD = DE$ and $BC = CE$,

$AD + BC = CD = 12$ and $AB = 8$. So the area is $\frac{1}{2}(8)(12) = 48$.

The area of the semicircle is 8π, so the area desired is **48 - 8π.**

T3. One method: Extend $\overline{AM}$ past M to D where MD = 7. Then ABDC is a parallelogram and triangle ABC has half its area. So is ABD, which is a triangle with side lengths 13, 14, and 15, whose area is 84 (e.g. by Hero's Formula). So triangle ABC also has area 84.
Alternate solution: let MB = MC = x. Since $\overline{AM}$ is a median, triangles ABM and ACM have the same area. Compute the area using Hero's Formula for each and set them equal. Solve to get x and find the area.

Alternate solution yet another: Use law of cosines twice.

T4. 3 Down is either 100 or 121. 1 Across is a cube with tens digit 1 so it is 4913. 4 Down ends in 3 and its middle digit is 0 or 6 since it's even and divisible by 3. 2 Down is either 900 or 961. 1 Down is 441 or 484 (can't be 400 since the across numbers can't start with 0). 6 Across can't start with 1 since 1 and 0 are the second and third digits in some order. So 1 Down is 484. If 2 Down is 961, then 3 Down must be 100 so the last digit of 2 across must repeat either 6 or 0. So 2 Down is 900. Thus 3 Down is 900 and 4 Down is 363.

$$\begin{bmatrix} \mathbf{4} & \mathbf{9} & \mathbf{1} & \mathbf{3} \\ \mathbf{8} & \mathbf{0} & \mathbf{2} & \mathbf{6} \\ \mathbf{4} & \mathbf{0} & \mathbf{1} & \mathbf{3} \end{bmatrix}$$

T5. Since $D(n) = 3$, n must be of the form $3 \cdot 2^n$ with $n < 1000$. So n = 0, 1, ...,8
and there are **9** such values.

T6. In the first 6 games, each team won three in any order. So the probability is given by
$\left({}_6C_3\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 = \frac{20}{64} = \mathbf{\frac{5}{16}}$.