Nassau County Interscholastic Mathematics League

Solutions **2003-2004**

Contest # 5

19. The table looks like

p	q	r	$[p \wedge (q \veebar r)] \to q$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

So there are **7**.

20. Let x = $m\angle PRS = m\angle QRS$ and $y = m\angle PQS = m\angle RQS$.
In ΔPQR, $2x + 2y + 88 = 180 \to 2x + 2y = 92 \to x + y = 46$. Now in ΔSQR, $x + y + m\angle SQR = 180$. Substituting 46 for $x + y$ $46 + m\angle SQR = 180, m\angle SQR = \mathbf{134}$.

21. See the figure. In the right triangle, half the chord is 24, so the chord is **48**.

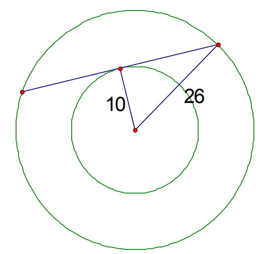

Figure for 21

22. Solving for y, we get $y = \dfrac{2x+1}{x-3} = 2 + \dfrac{7}{x-3}$.
For x and y to both be integers, $x - 3$ is a divisor of 7.
So x – 3 = 7, 1, -1, or –7. Then x = 4, 10, 2, or –4. Now substitute each x in to get y.
We get the ordered pairs $\mathbf{(4,9),(10,3),(2,\text{-}5),(\text{-}4,1)}$.

23. $g(-4) = -g(4)$, so their sum is 0. $g(0) = g(-0) = -g(0)$ so $g(0) = 0$.
Also $h(-2) = h(2)$. So the expression is $\dfrac{0 + 2h(2)}{h(2) + 0} = \dfrac{2h(2)}{h(2)} = \mathbf{2}$.

24. One method: $2(2\cos^2 x - 1) - 1 = 0,\ so\ \cos^2 x = \dfrac{3}{4}$. Now $\cos x = \pm\dfrac{\sqrt{3}}{2}$
and the angles are $\boldsymbol{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}}$.
Alternate method: solve for 2x over the interval 0 to 4π, then divide by 2.