Nassau County Interscholastic Mathematics League

Contest # 5 **Answers must be in simplest exact form, unless otherwise noted.** **2003-2004**

No Calculators

Problems 19-20 Time limit: 10 minutes.

19) A truth table is made for the logic expression $[p \wedge (q \veebar r)] \rightarrow q$, where symbols $\wedge$ represents "and", $\veebar$ represents "exclusive or", and $\rightarrow$ represents "implies". Of the eight cases for p, q, and r, for how many of them will the statement $[p \wedge (q \veebar r)] \rightarrow q$ be true? (Note, "exclusive or" $p \veebar q$ is true when either p or q is true but not both.)

20) Point S is in the interior of triangle PQR such that $\overline{RS}$ bisects $\angle PRS$ and $\overline{QS}$ bisects $\angle PQS$.
If $m\angle QPR = 88°$, find the degree-measure of $\angle QSR$.

Problems 21-22 Time limit: 10 minutes.

21) (a classic) The radii of two concentric circles are 10 and 26. Find the length of a chord of the larger circle which is tangent to the smaller circle.

22) Find all ordered pairs of integers (x, y) such that $1 + 2x + 3y = xy$.

Problems 23-24 Time limit: 10 minutes.

23) A function f is even if $f(-x) = f(x)$ for all x. A function f is odd if $f(-x) = -f(x)$ for all x. Suppose that function g is odd and function h is even. Completely simplify the expression $\dfrac{g(-4)+h(-2)+h(2)+g(4)}{h(2)+g(0)}$.

24) Find all exact real numbers x, with $0 \le x < 2\pi$, such that $2\cos(2x) - 1 = 0$.

Answers:	19)	7	20)	134
	21)	48	22)	$(4,9),(10,3),(2,-5),(-4,1)$
	23)	2	24)	$\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$