

Solutions
Contest #1

1. Multiply by y to get $x + 5xy = 6y + 1$, then $x - 1 = y(6 - 5x)$, so $y = \frac{x-1}{6-5x}$ or $y = \frac{1-x}{5x-6}$.
2. The ratio of the diameters is 2:3, so the ratio of the areas is 4:9. $\frac{4}{9} = \frac{12.40}{x}$, and $x = \$27.90$.
3. Any number starting with 4 works, and there are $\frac{5!}{2!2!} = 30$ of those. Also, 343212 works as does 343221. There are 32 of them.
4. Any one where p is true works; that's 4. If p is false, q must be true and r false; that's one more. There are 5.
5. To be geometric, the ratios of successive terms are equal, so $\frac{x-3}{x-1} = \frac{x-1}{3x-7}$, and so $3x^2 - 16x + 21 = x^2 - 2x + 1$, thus $2x^2 - 14x + 20 = 0$, then $x^2 - 7x + 10 = 0$. Solving, $x = 5$ or $x = 2$. If $x = 2$, then we get $-1, 1, -1, 1, \dots$. If $x = 5$, it's $2, 4, 8, 16, \dots$. The fourth term is either 1 or 16.
6. Let the sides be x and y , so $xy = 64$ and $x^2 + y^2 = 196$. Since $2xy = 128$, $x^2 + 2xy + y^2 = 324$. Now, $(x + y)^2 = 324$, and $x + y = 18$. The perimeter is $2(x + y) = 36$.