Nassau County Interscholastic Mathematics League

Solutions **2004-2005**

Contest #5

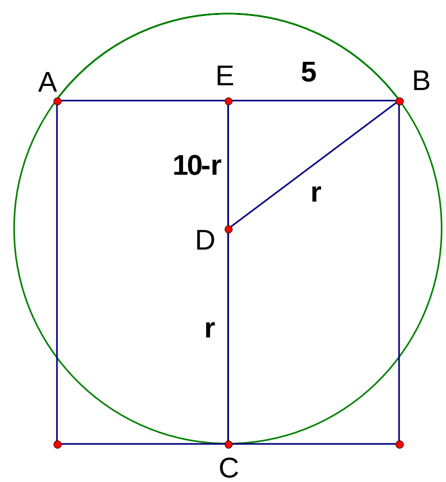
19. There are 210 taking both math and science, so there are 385 - 210 = 175 taking math, but not science, and 240 – 210 = 30 taking science, but not math. This accounts for 210 + 175 + 30 = 415 so far. So the number taking neither is 430 – 415 = 15. Thus the probability is $\frac{15}{430}=\frac{3}{86}$.

20. Method 1: Find the equations of any two medians and solve the system.

Method 2: the centroid is the average of the vertices, so the coordinates are $\left(\frac{1+2+9}{3},\frac{4+7+(-2)}{3}\right)=(4,3)$.

Method 3: the midpoint of one of the sides is $(5,1)$ and the point is $\frac{1}{3}$ of the way from there to the opposite vertex, $(2,7)$. That gets us to $(4,3)$.

Method 4: The three medians of a triangle are concurrent at a point which is $\frac{2}{3}$ the distance from any vertex.



21. In the figure shown, D is the center of the circle with radius r. Use the Pythagorean Theorem on right triangle DEB to get $(10-r)^2+5^2=r^2$, and $r=\frac{25}{4}$.Or use the power theorem (also called the intersecting chord theorem) . AE=5 and E to the circle is x so $10x=25, x=2.5$. The diameter is $\frac{25}{2}$ and the radius is $\frac{25}{4}$.

22. The equation simplifies to $y=\frac{(2x+1)^2(x+1)}{(3x-4)^3}$ So the only vertical asymptote is $x=\frac{4}{3}$ and since the numerator and denominator have the same degree, there is a horizontal asymptote, which is $y=\frac{4}{27}$. $x\neq-\frac{1}{2}, x\neq\frac{4}{3}, x\neq-1$.

23. The region is one-sixth of a circle of radius 6. So the area of $\frac{1}{6}\cdot 36\pi=6\pi$

24. Method 1: $y=\frac{92-5x}{7}=13+\frac{1-5x}{7}$ and we are dealing with integers so $1-5x$ must be a multiple of 7. So $1-5x=\ldots,-14,-7,0,7,14,21,\ldots$ and for each x we can compute the value of y. But we only take values where x and y are both positive. Those which work are (17, 1), (10, 6), and (3, 11).

Method 2: Work mod 5 and we get $2y \equiv 2 \pmod 5$, so $y \equiv 1 \pmod 5$. So y = 1, 6, 11, ... and we check to find x values that fit and are positive.