

Nassau County Interscholastic Mathematics League

Contest 5

Answers must be in simplest exact form unless otherwise specified. 2006–2007
No calculators

Problems 25-26. Time limit: 10 minutes

#25. If $3 \leq x \leq 8$ and $2 \leq y \leq 5$, compute the difference between the greatest possible value for the average of $\frac{1}{x}$ and $\frac{1}{y}$ and the least possible value of that average.

#26. Thirty double-scoop ice cream cones were sold in one hour at the Stroud Mall. If 17 cones contained at least one scoop of vanilla, 9 cones contained at least one scoop of chocolate, and 7 cones contained one scoop of each, what is the total number of cones which contained neither chocolate nor vanilla?

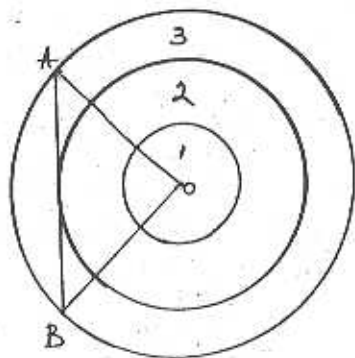
Problems 27-28. Time limit: 10 minutes

#27. Joan caught exactly 4 times as many fish as Ruth and exactly 6 times as many fish as Ahab. If, altogether, the three of them caught fewer than 150 fish, determine the greatest number of fish that Joan could have caught.

#28. Compute the least positive degree-measure of x for which $8 \sin x \cos^5 x - 8 \sin^5 x \cos x = \sqrt{2}$.

Problems 29-30. Time limit: 12 minutes

#29. Three circles are concentric at point O as shown in the diagram below. The circle with the second largest area will be referred to as the “middle” circle. Region 1 is the area of the smallest circle. The areas of Regions 1, 2, and 3 are equal. The radius of the largest circle is 15 and chord \overline{AB} of the largest circle is tangent to the middle circle. Compute the exact area of $\triangle AOB$. [Note: The figure shown is representative, but not drawn to scale.]



#30. Compute the total number of integers for which $|x^2 - 10x - 56| \neq x^2 - 10x - 56$.

Answers:

#25. $\frac{61}{240}$

#26. 11

#27. 96

#28. $\frac{1}{32}$

#29. $75\sqrt{2}$

#30. 17