Nassau County Interscholastic Mathematics League

Solutions, Contest 5

#25. $\frac{1}{8} \le \frac{1}{x} \le \frac{1}{3}$ and $\frac{1}{5} \le \frac{1}{y} \le \frac{1}{2}$. The greatest possible average is $\frac{\frac{1}{2}+\frac{1}{3}}{2} = \frac{5}{12}$. The least possible average is $\frac{\frac{1}{8}+\frac{1}{5}}{2} = \frac{13}{80}$. The difference between these two averages is $\frac{61}{240}$.

#26. $30 - (17 + 9 - 7) = 11$.

#27. Let x = the number of fish that Joan caught.

Then $\frac{x}{4} + \frac{x}{6} + x < 150; 3x + 2x + 12x < 1800;\ x < 105\frac{15}{17}$. Since x must be divisible by 12, the greatest value of x is 96.

#28. $8 \sin x \cos x(\cos^4 x - \sin^4 x) = \sqrt{2}$; $4 \sin 2x(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \sqrt{2}$;

$4 \sin 2x \cos 2x = \sqrt{2}$; $2 \sin 4x = \sqrt{2}$; $\sin 4x = \frac{\sqrt{2}}{2}$; $4x = 45$; $x = 11.25$.

#29. The area of the smallest circle is 75π; the area of the middle circle is 150π The radius of the middle circle, $5\sqrt{6}$, is the length of the altitude to side $\overline{AB}$ of $\triangle$ AOB. By the Pythagorean Theorem, $\frac{1}{2}AB = \sqrt{15^2 - (5\sqrt{6})^2} = 5\sqrt{3}$.

Therefore, the area of triangle AOB is $5\sqrt{3} \cdot 5\sqrt{6} = 25\sqrt{18} = 75\sqrt{2}$.

#30. If the absolute value of a quantity is not equal to itself, then the quantity is neither positive nor zero. It must be negative. So, $x^2 - 10x - 56 < 0$; $(x + 4)(x - 14) < 0$; $-4 < x < 14$. The number of integers in this interval is 17.
