SAMPLE SOLUTIONS, Contest #2

7. Answer: 2

$72 = 2^3 \cdot 3^2$ and $96 = 2^5 \cdot 3$; $\dfrac{2^{459}\cdot 3^{306}}{2^{245}\cdot 3^{49}} = 2^{214}\cdot 3^{257}$.
$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ (ends in 6), and $2^5 = 32$ (ends in 2). The cycle then starts to repeat itself with respect to what the units' digit will be. So, the units digit for 2^{214} is 4.
$3^1 = 3$, $3^2 = 9$, $3^3 = 27$ (ends in 7), $23^4 = 81$ (ends in 1), and $3^5 = 243$ (ends in 3). The cycle then starts to repeat itself with respect to what the units' digit will be. So, the units digit for 3^{257} is 3.
The units' digit of $2^{214}\cdot 3^{257}$ is the units' digit of the product of 3 and 4.

8. Answer: 50

Let x = the degree-measure of the central angle of the sector of the larger circle

$\dfrac{112.5}{360}\cdot 64\pi = \dfrac{x}{360}\cdot 144\pi$ **; $x = 50$**

9. Answer: 20

$w^2 - w = 2.5$; $(w^2 - w)^2 = w^4 - 2w^3 + w^2 = 6.25$;
$w^4 - 2w^3 + w^2 + 6(w^2 - w) - 1.25 = 6.25 + 15 - 1.25 = 20$.

10. Answer: 420

Lel l be the longer dimension, w be the shorter dimension, and d be the length of the diagonal. Then, $2(l + w) = 2d + 20$ or $l + w = d + 10$ (1); $l - w = d - 14$ (2). If you subtract equation (2) from equation (1), you get $w = 1^2$ and $d = l + 2$. By the Pythagorean theorem, $d^2 = l^2 + w^2 \Rightarrow (l + 2)^2 = l^2 + 144$ and $l = 35$. The area of the rectangle is (35)(12) or 420.

11. Answer: 0.16 or $\dfrac{4}{25}$

Let the numbers be a and b. Then $a + b = 6$ and $a\,b = 10$.

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{a^2+b^2}{a^2b^2}=\frac{(a+b)^2-2ab}{(ab)^2}=\frac{36-20}{100}=\frac{16}{100}=\frac{4}{25}=0.16.$$

12. Answer: (4, 281)

The slope of the line given by $10x + 7y = 2007$ is $-\dfrac{10}{7}$, meaning for every rise of 10 units in the y-coordinate, there is a corresponding fall of 7 units in the x-coordinate. $200 \div 7 = 28R4$.
$28(10) + 1 = 281$