

SAMPLE SOLUTIONS, Contest #4

19. Answer: 37

Let  $T$  and  $B$  be Ted's and Bill's present age, respectively.

Then,  $T = 3(B - 6)$  and  $T + 10 + B + 10 = 5B - 13$ . By substitution,  $T = 27$ . and  $T + 10 = 37$ .

20. Answer: 4

$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$ , after which the cycle repeats itself with respect to the units' digit. Thus,  $3^{25}$  will have a units' digit of 3.

$8^1 = 8, 8^2 = 64, 8^3 = 512, 8^4 = 4096$ , etc. So,  $8^{31}$  will have a units' digit of 2.

$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401$ , etc. So,  $7^{12}$  will have a units' digit of 1.

The units' digit for  $3^{25} + 8^{31} - 7^{12}$  is  $3 + 2 - 1 = 4$ . When the quantity is divided by 10, the remainder will also be 4.

21. Answer: 202.5

Let  $x$  be the distance from city B to city C.

Then,  $\frac{x}{45} + \frac{300-x}{65} = 6$ ;  $65x + 13500 - 45x = 17550$ ;  $x = 202.5$

22. Answer: 5.5

Let  $x\%$  be the desired annual interest rate.

$$29000 \left(1 + \frac{x}{365}\right)^{270} = 30000 \left(1 + \frac{0.028}{365}\right)^{90}; \left(1 + \frac{x}{365}\right)^{270} = \frac{30}{29} \left(1 + \frac{0.028}{365}\right)^{90};$$

$$270 \log \left(1 + \frac{x}{365}\right) = \log 30 - \log 29 + 90 \log \left(1 + \frac{0.028}{365}\right); \log \left(1 + \frac{x}{365}\right) = 0.00006563540192;$$

$$1 + \frac{x}{365} = 10^{0.00006563540192}; x = 5.51670194$$

23. Answer:  $\frac{9}{4}\pi$

Let  $r$  be the radius of circle O. Draw radii  $\overline{OA}$  and  $\overline{OB}$ .  $\triangle AOP \sim \triangle BRP$ .

$\frac{OA}{RB} = \frac{PA}{PB} = \frac{2}{8} = \frac{1}{4}$ ;  $\therefore RB = 4r$ .  $\overline{OA} \perp \overline{PB}$  and  $\overline{RB} \perp \overline{PB}$ . From O, draw a segment perpendicular to  $\overline{RB}$ , meeting  $\overline{RB}$  at S. In rt  $\triangle AOP$ ,  $OS = 6$ ,  $RS = 3r$ , and  $OR = 5r$ .

$$(5r)^2 = (3r)^2 + 36; r^2 = \frac{9}{4}; \text{ the area of circle O is } \frac{9}{4}\pi.$$

24. Answer: 500 [Accept 500 mph.]

Let  $x$  be the speed in mph of the plane in still air on each leg of the trip.

$$\text{Then, } \frac{6240}{x-20} = \frac{6240}{x+20} + 1; 6240(x+20) = 6240(x-20) + (x+20)(x-20); x^2 = 250000; x = 500$$