

SAMPLE SOLUTIONS, Contest #5

25. Answer:  $22\sqrt{5}$

Let  $A$  represent the area of the smaller rectangle. Then  $A + A + 225 = 405$  and  $A = 90$ . One side of the rectangle is  $\sqrt{405} = 9\sqrt{5}$ . The other side of the rectangle is  $\frac{90}{9\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$ . The perimeter of the rectangle is  $2(9\sqrt{5} + 2\sqrt{5}) = 22\sqrt{5}$ .

26. Answer:  $\left\{ \pm 2, \pm \frac{3}{2} \right\}$

Rearranging the terms so they are in standard form,  $16x^6 - 64x^4 - 81x^2 + 324 = 0$ .

Factoring by grouping,  $16x^4(x^2 - 4) - 81(x^2 - 4) = 0$ ;  $(x^2 - 4)(16x^4 - 81) = 0$ ;

$(x^2 - 4)(4x^2 - 9)(4x^2 + 9) = 0$ ;  $(x - 2)(x + 2)(2x - 3)(2x + 3)(4x^2 + 9) = 0$ .

The real roots are  $\pm 2$  and  $\pm \frac{3}{2}$ . The imaginary roots are  $\pm \frac{3}{2}i$ .

27. Answer: 512

$$25^{1/4} + x^{1/3} = \frac{59}{8 - \sqrt{5}} \cdot \frac{8 + \sqrt{5}}{8 + \sqrt{5}} = 8 + \sqrt{5}; \quad (5^2)^{1/4} + \sqrt[3]{x} = 8 + \sqrt{5}; \quad \sqrt[3]{x} = 8 \text{ and } x = 512.$$

28. Answer: 11:40 pm

Let the height of each original candle be 1 and let  $x$  be the number of hours that the second candle is burning,  $0 < x \leq 4$ .  $1 - \frac{x}{5} = 4\left(1 - \frac{x+1}{5}\right)$ ;  $5 - x = 20 - 4x - 4$ ;  $x = \frac{11}{3}$ . The second candle burns  $3\frac{2}{3}$  hours, that is, 3 hours and 40 minutes.

29. Answer: 10

$\frac{9 \cdot 8}{2} + \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = \frac{x(x-1)(x-2)}{3 \cdot 2}$ ;  $x(x-1)(x-2) = 720$ . The three positive consecutive integers with a product of 720 from least to greatest are 8, 9, and 10.

30. Answer: 15

$$\cos 2\theta = 1 - 2\sin^2 \theta; \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{2 - \sqrt{3}}{4}; \quad 4 - 4 \cos 2\theta = 4 - 2\sqrt{3}; \quad \cos 2\theta = \frac{\sqrt{3}}{2};$$

$$2\theta = 30^\circ.$$